

Exercise session 7

Algebraic Topology 2024-2025

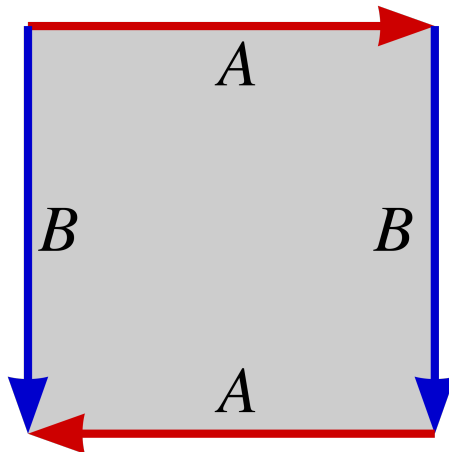
25 March, 2025

Exercise 1

Calculate the fundamental group of the torus using the Van Kampen theorem.

Exercise 2

Recall that the Klein Bottle is defined as by identifying the sides of a square as depicted in the picture.



Compute the fundamental group of the Klein bottle minus a point.

Exercise 3

Compute the fundamental group of the Klein bottle. Hint: the previous two exercises show the way. You may not be able to give a nice description, one in terms of generators and relations suffices.

Exercise 4

Compute the fundamental group of the real projective space $\mathbb{R}P^2$. Hint: recall that $\mathbb{R}P^2$ is homeomorphic to the quotient

$$S^2/\{x \sim -x\};$$

this, in turn, is homeomorphic to the quotient of

$$D^2/\{x \sim -x\}, \text{ for } x \in \partial D^2 \cong S^1.$$

Exercise 5

Compute the fundamental group of the real projective space $\mathbb{R}P^3$. Hint: recall that $\mathbb{R}P^3$ is homeomorphic to the quotient

$$D^3/\{x \sim -x\}, \text{ for } x \in \partial D^3 \cong S^2;$$

generalize this to compute the fundamental group of $\mathbb{R}P^n$ for all n .