Let's use the Mayer-Vietoris sequence to compute the homology of spheres. First, the case of S^1. Cover S^1 with two opens U, V which are both contractible and such that the intersection is homotopy equivalent to two points (note that there is no connectivity hypothesis in the Mayer-Vietoris sequence!). S^1 is arc connected, so $H_0(S^1) = R$. It's easy to show that all the homology groups for $n \geq 2$ vanish: we have an exact sequence

$$0 = H^n(U) \oplus H^n(V) \to H^n(S^1) \to H^n(U \cap V) = 0$$

so $H^n(S^1) = 0$ for $n \geq 2$. The case $n = 1$ is slightly more complicated, because in this case the exact sequence is

$$0 \to H^1(S^1) \to H^0(U \cap V) \to H^0(U) \oplus H^0(V) \to H^0(X) \to 0$$

which concretely is an exact sequence

$$0 \to H^1(S^1) \to R \oplus R \xrightarrow{f} \mathbb{Z} \oplus \mathbb{Z} \to \mathbb{Z} \to 0$$

so that $H^1(S^1)$ is isomorphic to the kernel of f. Unraveling the definitions, one finds that $f(x, y) = (x+y, x+y)$ (check this!) and therefore $H_1(S^1) = \mathbb{Z}$.

This generalizes easily to higher dimensions: consider the sphere S^n for $n \geq 2$, and cover it by two opens U, V which are contractible and whose intersection is homotopy equivalent to the sphere S^{n-1} (for example, you can take U and V to be the whole sphere minus the north and south pole respectively). Then the Mayer-Vietoris sequence gives, for $k \geq 1$, an exact sequence

$$0 \to H_k(S^n) \to H_{k-1}(S_{n-1}) \to 0$$

so $H_k(S^n) \cong H_{k-1}(S^{n-1})$ and hence, by induction $H_n(S^n) = \mathbb{Z}$ and $H_i(S^n) = 0$ for $i > n$. Similarly, one shows that $H_i(S^n) = 0$ for $0 < i < n$. Hence,

$$H_k(S^n) = \begin{cases} \mathbb{Z} & \text{for } k = 0 \text{ and } k = n; \\ 0 & \text{otherwise.} \end{cases}$$