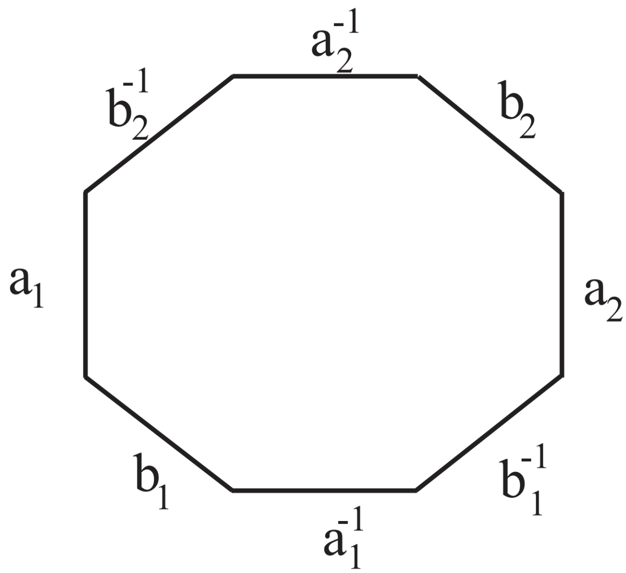
1. Let $X = S^4 - S^2 \subseteq \mathbb{R}^5$; compute $\pi_1(X)$ and $H_i(X, \mathbb{Z})$ for all i.

2. Recall the characterization of the torus T^2 as the quotient of a square obtained by identifying the opposite sides.

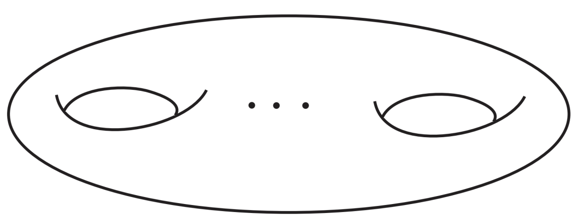
- Consider a regular polygon with $4g$ sides; denote its sides, ordered in a circular way,

$$a_1, b_1, a_1^{-1}, b_1^{-1}, a_2, b_2, \ldots, a_g^{-1}, b_g^{-1}.$$

Orientate the boundary by taking the sides denoted with a_i, b_i with the same orientation as the boundary and the ones denoted with a_i^{-1}, b_i^{-1} with the opposite one. Below is the case $g = 2$.



Denote with M_g the surface obtained by identifying a_i with a_i^{-1} and b_i with b_i^{-1}. This is called the genus g surface, or torus with g holes.



- Calculate $\pi_1(M_g)$ (as usual, by generators and relations).
- Compute the homology of M_g.

Some more space for the solution:

Cont.

3. Let $C = \{(x, y, z) \in \mathbb{R}^3 |\, x^2 + y^2 = 1\}$ be a cylinder, and let Y be C minus one point. Compute $\pi_1(Y)$ and $H_i(Y, \mathbb{Z})$ for all i.

4. Find an example of two topological spaces X, Y for which the group $H_1(X \times Y)$ is not isomorphic to $H_1(X) \times H_1(Y)$. Optional extra question: given topological spaces X, Y with the property that $H_1(X)$ and $H_1(Y)$ are free $\mathbb{Z}$-modules, can you write down (without proving it) a formula for the rank of $H_1(X \times Y)$?

The End.